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A borehole temperature during drilling in a fractured rock formation

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Abstract

Drilling in brittle crystalline rocks is often accompanied by a fluid loss through the finite number of the major fractures intercepting the borehole. These fractures affect the flow regime and temperature distributions in the borehole and rock formation. In this study, the problem of borehole temperature variation during drilling of the fractured rock is analyzed analytically by applying the approximate generalized integral-balance method. The model accounts for different flow regimes in the borehole, for different drilling velocities, for different locations of the major fractures intersecting the borehole, and for the thermal history of the borehole exploitation, which may include a finite number of circulation and shut-in periods. Normally the temperature fields in the well and surrounding rocks are calculated numerically by the finite difference and finite element methods or analytically, utilizing the Laplace-transform method. The formulae obtained by the Laplace-transform method are usually complex and require tedious numerical evaluations. Moreover, in the previous research the heat interactions of circulating fluid with the rock formation were treated assuming constant bore-face temperatures. In the present study the temperature field in the formation disturbed by the heat flow from the borehole is modeled by the heat conduction equation. The thermal interaction of the circulating fluid with the formation is approximated by utilizing the Newton law of cooling at the bore-face. The discrete sinks of fluid on the bore-face model the fluid loss in the borehole through the fractures. The heat conduction problem in the rock is solved analytically by the heat balance integral method. It can be proved theoretically that the approximate solution found by this method is accurate enough to model thermal interactions between the borehole fluid and the surrounding rocks. Due to its simplicity and accuracy, the derived solution is convenient for the geophysical practitioners and can be readily used, for instance, for predicting the equilibrium formation temperatures.

Keywords: Fluid loss; Borehole; Temperature; Heat flux; Bore-face; Fluid circulation; Integral-balance method

1. Introduction

A reliable assessment of thermal interaction between the borehole and the surrounding rock formation is of considerable interest in a number of geophysical

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Nomenclature

A, B constants which define the equilibrium temperature of the rock formation, $t_f(z^*) = Az^* + B$
a_i^d, a_i^a, b_i, a_i^d, c_i^a, c^d coefficients in Eqs. (9) and (10), which are defined by Eqs. (15)
Bi, Bi_i Biot numbers defined by Eqs. (7) and (17), respectively
c_r, c_L specific heat of the rock and fluid, respectively
$c_i^{(1)}$, $c_i^{(2)}$ constants of integration in Eqs. (32) and (33)
d_r, d_L, d_p thermal diffusivities of the formation, liquid and drilling pipe, respectively
d_i parameters defined by Eq. (34)
f non-dimensional geothermal gradient defined by Eqs. (16)
G flow rate in the drilling pipe
G_i the flow rate in the annulus between the fractures $(i - 1)$ and i
H depth of the borehole during drilling (a function of time)
H_0 initial depth of the borehole at the onset of the next drilling cycle
h^w heat transfer coefficient on the bore-face
h^d, h_i^d heat transfer coefficients on the inner and outer walls of the drilling pipe
h_i coefficient defined by Eqs. (17)
J_0, J_1 Bessel functions of the first kind of the order 0 and 1, respectively
q_i heat fluxes on the bore-face defined by Eqs. (22) and (25)
$\bar{q}_i$ function defined by Eq. (36)
k_r, k_L, k_p thermal conductivities of the formation and fluid and drilling pipe, respectively
l radius of thermal influence
N number of fractures intercepting the well
r_0 external radius of the drilling pipe
r_w radius of the borehole
r, z non-dimensional cylindrical coordinates
T_r non-dimensional temperature of the formation during drilling defined by Eqs. (7)
T_i^a, T_i^d non-dimensional temperature of fluid in the annulus and drilling pipe defined by Eqs. (16)
T_{in} non-dimensional temperature injected fluid defined by Eqs. (16)
t^a, t^d temperatures in the annulus and drilling pipe, respectively
t_r temperature of formation during fluid circulation
t_{in} temperature of injected fluid
t_f equilibrium temperature of formation
t_0 temperature in formation defined by Eq. (8)
U_0, U^a, U^d non-dimensional temperatures defined by Eqs. (42)
V drilling velocity
v^d, v_i^a mean fluid velocities in the drilling pipe and in the annulus, respectively
z_i location of the ith fracture intercepting the well
Y_0, Y_1 Bessel functions of the second kind of the order 0 and 1, respectively
δ thickness of the drilling pipe wall
η function defined by Eqs. (31)
$\lambda_i^{(1)}$, $\lambda_i^{(2)}$ parameters defined by Eq. (35)
$\bar{\lambda}_i^{(1)}$, $\bar{\lambda}_i^{(2)}$ parameters defined by Eq. (41)
μ fluid viscosity
ρ_r, ρ_L densities of rock and liquid, respectively
τ time
τ_H^* time, required to drill a well to the depth H

Superscripts
a annulus
d drilling pipe
w wall of the borehole
* dimensional quantities

Subscripts
d drilling pipe
i ith section in the borehole between fracture $(i - 1)$ and i
L liquid
m mean value
r rock
w wall of the borehole

applications. The following applications are worth mentioning: (i) interpretation of electric logs and estimation of the formation temperatures from well logs, which requires knowledge of temperature disturbances in the formation produced by circulating fluid during drilling [1–4]; (ii) optimal design of the drilling bit cooling system within the high-temperature formation [5] requires assessment of the heat either delivered from the high temperature rocks to the drilling bit or transmitted to the formation from the circulating fluid; (iii) developing the new technologies and methods in the area of geothermal energy production [6–8]. Normally, temperature fields in the well and surrounding rocks are calculated numerically [1,2,9–12] by using a finite difference method.

The exact analytical solutions of the heat conduction problem in the rock formation (obtained by Laplace transformation in [3,13]) are rather complex and require tedious numerical evaluations. Therefore, they are not very convenient for an engineering estimation. In a number of previous publications the heat interaction of the circulating fluid with the formation was treated under the condition of constant bore-face temperature [14–20]. Based on the latter approach and employing some additional simplifying assumptions, several simple analytical formulae for the temperature distribution in the rock formation and for the heat flux on the bore-face were proposed [17–21]. However, the assumption of constant bore-face temperature is not realistic and, therefore, the temperature on the bore-face should be treated as an unknown function of time and axial coordinate, z, in the mathematical modeling. For this reason, the previously obtained solutions have a limited range of practical applicability and can be used only in the case of highly intensive heat transfer between the circulating fluid and surrounding media. In the present study Newton's model of convective heat transfer on the bore-face is employed. Carslaw and Jaeger [22] found an exact analytical solution of the problem that models heat conduction in the rock formation, but their solution is rather complex.

In order to avoid the complexity of the exact solution and to obtain a much simpler solution convenient for geophysical studies, an approximate analytical integral-heat-balance method proposed by Goodman [23] and later improved by Volkov et al. [24] is utilized in this paper. This method was successfully applied by Fomin et al. [25] for solving the problem of a moving heat source within the borehole and also by Chugunov et al. [26] for computing heat fluxes in the case of non-homogeneous domain. A simplified solution provided by this method could be also beneficial for its further incorporation into the model of heat and mass transfer processes during drilling and exploitation.

Luheshi [1] and Shen and Beck [3] investigated the influence of the circulating fluid loss through the permeable wall of the well. These investigations demonstrate the drastic effect of the fluid loss during the production stage on the borehole temperature stabilization at the shut-in period. In previous studies [1,3,27], the effect of a circulating fluid loss in a surrounding rock through the permeable bore-face was investigated under the assumption that the fluid penetrates in the formation uniformly along the well's wall. Apparently, this type of uniform filtration through the bore-face can occur only for a sedimentary porous formation, whereas for crystalline fractured rocks this assumption is not valid. In the present model, applicable for drilling in brittle fractured rocks, we assume that the fluid loss occurs predominantly through the major fractures intercepting the well and that the number and location of the permeable fractures is determined by the preliminary geophysical investigations. This approach is consistent with field observations that verify the discrete character of the subsurface fractures distribution [28].

2. System model and analysis

During drilling, water is injected through the inner (drilling) pipe and returns up to the surface along an annulus (a gap between the drilling pipe and the wall of the borehole). The circulation of fluid is used for cooling a drilling bit and for removing debris from the borehole. Schematically, fluid circulation in the borehole is illustrated in Fig. 1. The borehole fluid is assumed to be well stirred laterally. This allows the analysis to be limited to the cross-sectional averages of the fluid temperatures in the drilling pipe, $t^{\mathrm{d}}(z^*,\tau^*)$, and in the annulus, $t^{\mathrm{a}}(z^*,\tau^*)$, as functions of the axial coordinate and time. This is analogous to the assumption of Shen and Beck [3] that the borehole fluid remains a perfect conductor in a radial direction. Circulation of the fluid in the borehole during drilling or during the production stage of the borehole exploitation disturbs the initial equilibrium temperature of the rock formation, t_{f}, which is often highly non-linear, especially in the regions of the strong volcanic activity [5]. However, in a great number of situations of practical interest the geothermal gradient, A, within rock formation can be approximately assumed constant [4,12,17,20] and, therefore, the equilibrium temperature can be approximated by a linear function of depth, $t_{\mathrm{f}}(z^*) = Az^* + B$. The heat transfer on the bore-face, $r^* = r_{\mathrm{w}}$, is modeled by the Newton's law, $-k_{\mathrm{r}}\partial t_{\mathrm{r}}/\partial r^* = h^{\mathrm{w}}(t^{\mathrm{a}} - t_{\mathrm{r}})$, where t_{r} is the formation temperature disturbed by the fluid circulation. This model accounts for the influence of the flow regime in the borehole, since the heat transfer coefficient, h^{w}, differs to an order of magnitude for the laminar, transient to turbulent, and fully-developed turbulent flow regimes. The depth of the borehole H during drilling increases with time. Assuming the drilling velocity, V, to be constant, the variation of the borehole depth with time can be approximated by equation $H = H_0 + V\tau_H^*$, where H_0 is the initial depth of the borehole and τ_H^* is the time required to drill a well of the depth H. Since the drilling speed is relatively high, the formation temperature below the borehole bottom remains practically undisturbed. In other words, below the drilling bit where $z^* > H_0$ the formation temperature $t_{\mathrm{r}} \approx t_{\mathrm{f}}$. The time required for drilling bit to reach the depth z^* can be computed from the following equation

$$\tau^* = (z^* - H_0)/V. \tag{1}$$

Since the radial temperature gradients are typically 100–1000 times greater than temperature gradients in the vertical direction [1,3], the derivatives of temperature

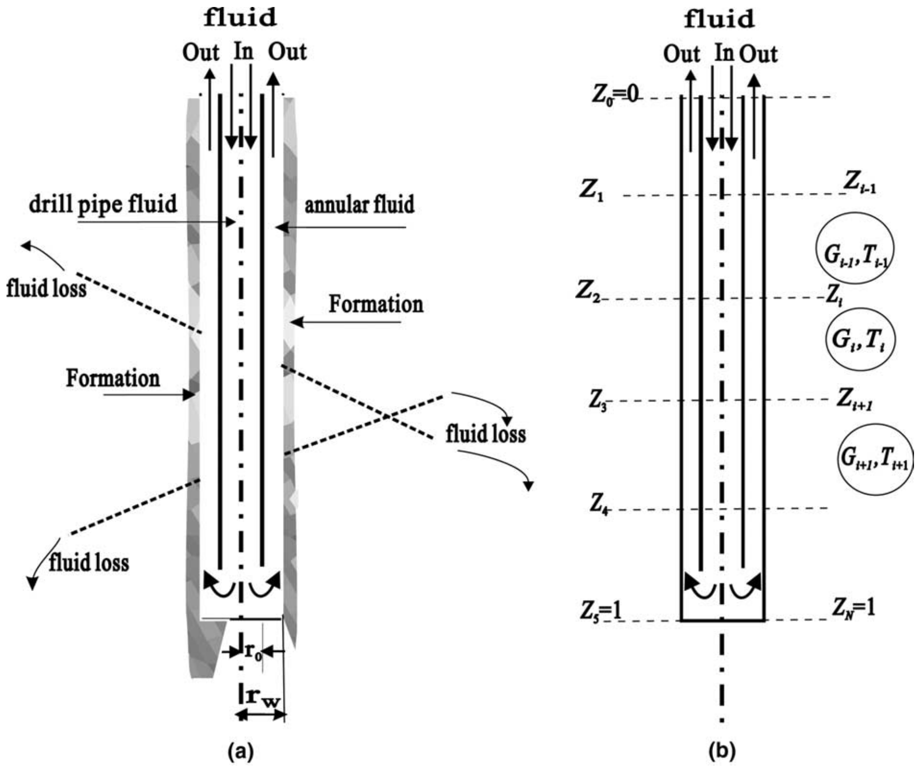

Fig. 1. Schematic sketch of the model. (a) Physical model of the flow regime and hypothetical borehole—fracture interception; (b) conventional division of the well into subsections with different flow rates in the annulus.

with respect to z^* can be neglected in the governing equations and the temperature distribution, t_r, in the surrounding rock during the fluid-circulation period in cylindrical coordinates (r, z) can be described by the following non-dimensional mathematical models:

Model 1. In the rock around the preliminary completed upper part of the borehole (in the interval $0 < z^* < H_0$), for $1 < r < \infty$, $\tau > 0$, the temperature distribution is governed by the following equation:

$$\frac{\partial T_r}{\partial \tau} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_r}{\partial r}\right), \tag{2}$$

with the following boundary conditions

$$r = 1, \quad -\partial T_r/\partial r = Bi[T^a(z,\tau) - T_r], \tag{3}$$

$$\lim_{r\to\infty} T_r < \infty, \tag{4}$$

and the following initial condition

$$\tau = 0, \quad T_r = 0. \tag{5}$$

Model 2. Temperature field in the rock around the lower part of the borehole for $\tau^* > (z^* - H_0)/V$, where drilling takes place (the interval $H_0 < z^* < H$), also can be modeled by Eq. (2) and boundary conditions (3) and (4), whereas due to relationship (1), the initial condition (5) should be replaced with

$$\tau = \frac{d_r H(z - H_0/H)}{r_w^2 V}, \quad T_r = 0. \tag{6}$$

The non-dimensional variables in Eqs. (2)–(6) are defined by the following relationships:

$$r = \frac{r^*}{r_w}, \quad z = \frac{z^*}{H}, \quad Bi = \frac{h_w r_w}{k_r}, \quad \tau = \frac{\tau^* d_r}{r_w^2},$$
$$T_r = \frac{t_r - t_0}{AH}, \quad T^a = \frac{t^a - \left(t_0 - \frac{1}{Bi}\frac{\partial t_0}{\partial r}\right)\big|_{r=1}}{AH}, \tag{7}$$

where temperature in the formation at the onset of circulation, $t_0(r^*, z^*)$, satisfies Eq. (2). In a particular case it may be equal to the equilibrium formation temperature,

$$t_0 = t_f(z^*) = Az^* + B. \tag{8}$$

Fluid flow and heat transfer within the borehole are greatly affected by the fluid loss through the natural rock fractures intersecting with the well. In a brittle rock a substantial portion of the fluid in the borehole leaks away into surrounding rock masses through a finite number of major intercepting fractures. Conventionally, it can be assumed that in total N major fractures intercept the well. Schematically it is illustrated in Fig. 1. It is assumed that the location of each ith fracture is known and defined by its axial coordinate, $z^* = z_i^*$. Since the flow rates G_i and G_{i+1} in the annulus above and below the ith fracture, respectively, are different (due to the fluid loss through this fracture), the heat transfer rates and temperatures are also different. Within the ith section of the borehole between two fractures located at coordinates $z^* = z_{i-1}^*$ and $z^* = z_i^*$, the flow rate in the annulus is denoted by G_i, the temperatures in the drilling tube by t_i^{d} and in the annulus by t_i^{a}. Then heat transfer in the borehole divided into $N + 1$ sections can be modeled by the following non-dimensional equations:

$$c^{\mathrm{d}}\frac{\partial T_i^{\mathrm{d}}}{\partial \tau} + \frac{\partial T_i^{\mathrm{d}}}{\partial z} = a_i^{\mathrm{d}}(T_i^{\mathrm{a}} - T_i^{\mathrm{d}}) + f(z), \tag{9}$$

$$c^{\mathrm{a}}\frac{\partial T_i^{\mathrm{a}}}{\partial \tau} - \frac{\partial T_i^{\mathrm{a}}}{\partial z} = -a_i^{\mathrm{a}}(T_i^{\mathrm{a}} - T_i^{\mathrm{d}}) - b_i(T_i^{\mathrm{a}} - T_{\mathrm{w}}) - f(z), \tag{10}$$

where $i = 1, 2, \ldots, N + 1$. These equations must be solved subject to the following boundary and initial conditions:

$$z = z_0 = 0; \quad T_1^{\mathrm{d}} = T_{\mathrm{in}}, \tag{11}$$

$$z = z_i; \quad T_i^{\mathrm{a}} = T_{i+1}^{\mathrm{a}}, \quad T_i^{\mathrm{d}} = T_{i+1}^{\mathrm{d}}, \quad (i = 1, 2, \ldots, N), \tag{12}$$

$$z = z_{N+1} = 1; \quad T_{N+1}^{\mathrm{a}} = T_{N+1}^{\mathrm{d}}, \tag{13}$$

$$\tau = 0; \quad T_i^{\mathrm{a}} = T_i^{\mathrm{d}} = 0, \quad (i = 1, 2, \ldots, N + 1). \tag{14}$$

where $z_{N+1} = 1$ and $z_0 = 0$ correspond to the bottom and the mouth of the borehole, respectively. Hence, in the general case of N fractures intersecting the borehole, the heat transfer process is governed by $2(N + 1)$ Eqs. (9) and (10) defined in the intervals (z_{i-1}, z_i), where $i = 1, 2, \ldots, N + 1$. In the particular case, when there are no fractures and no fluid loss in the formation, heat transfer in the borehole can also be described by Eqs. (9) and (10) and boundary conditions (11), (3) and (14) (condition (12) should be dropped), where N should be set to zero and i should be set to unity.

In Eqs. (9)–(14) the non-dimensional variables and parameters are defined as follows:

$$a_i^{\mathrm{d}} = \frac{2\pi r_0 H h_i}{c_{\mathrm{L}} G}, \quad a_i^{\mathrm{a}} = \frac{2\pi r_0 H h_i}{c_{\mathrm{L}} G_i}, \quad b_i = \frac{2\pi k_{\mathrm{w}} H Bi_i}{c_{\mathrm{L}} G_i},$$
$$c^{\mathrm{d}} = \frac{H d_{\mathrm{r}}}{r_{\mathrm{w}}^2 v^{\mathrm{d}}}, \quad c_i^{\mathrm{a}} = \frac{H d_{\mathrm{r}}}{r_{\mathrm{w}}^2 v_i^{\mathrm{a}}}, \quad z_i = \frac{z_i^*}{H}, \tag{15}$$

$$T_i^{\mathrm{a}} = \frac{t_i^{\mathrm{a}} - t_0}{AH}, \quad T_i^{\mathrm{d}} = \frac{t_i^{\mathrm{d}} - t_0}{AH}, \quad T_{\mathrm{w}} = T_{\mathrm{r}}|_{r=1},$$
$$T_{\mathrm{in}} = \frac{t_{\mathrm{in}} - t_0|_{z=0}}{AH}, \quad f = -\frac{1}{A}\frac{\partial t_0}{\partial z^*} \tag{16}$$

$$h_i = \frac{1}{[(1/h^{\mathrm{d}}) + (1/h_i^{\mathrm{a}}) + (\delta/k_{\mathrm{d}})]},$$
$$v_i^{\mathrm{a}} = \frac{G_i}{\pi(r_{\mathrm{w}}^2 - r_0^2)\rho_{\mathrm{L}}}, \quad v^{\mathrm{d}} = \frac{G}{\pi r_0^2 \rho_{\mathrm{L}}}, \quad Bi_i = \frac{r_{\mathrm{w}} h_i^{\mathrm{w}}}{k_{\mathrm{w}}}. \tag{17}$$

The values of the heat transfer coefficients h^{d}, h_i^{a}, and h_i^{w} depend on the flow regime in the borehole and thermophysical properties of the fluid. They can be calculated using experimentally obtained correlations, which are well documented and available in [29]

$$h^{\mathrm{d}} = 0.021(Re^{\mathrm{d}})^{0.8}(Pr_{\mathrm{L}})^{0.43}(Pr_{\mathrm{L}}/Pr_{\mathrm{d}})^{0.25}k_{\mathrm{L}}/(2r_0),$$
$$h_i^{\mathrm{w}} = 0.021(Re_i^{\mathrm{a}})^{0.8}(Pr_{\mathrm{L}})^{0.43}(Pr_{\mathrm{L}}/Pr_{\mathrm{w}})^{0.25}k_{\mathrm{L}}/(2(r_{\mathrm{w}} - r_0)) \tag{18}$$

where

$$Re^{\mathrm{d}} = \frac{\rho_{\mathrm{L}} v^{\mathrm{d}} 2r_0}{\mu}, \quad Re_i^{\mathrm{a}} = \frac{\rho_{\mathrm{L}} v_i^{\mathrm{a}} 2(r_{\mathrm{w}} - r_0)}{\mu},$$
$$Pr_{\mathrm{L}} = \frac{\mu}{\rho_{\mathrm{L}} d_{\mathrm{L}}}, \quad Pr_{\mathrm{d}} = \frac{\mu}{\rho_{\mathrm{d}} d_{\mathrm{d}}}, \quad Pr_{\mathrm{w}} = \frac{\mu}{\rho_{\mathrm{r}} d_{\mathrm{r}}}.$$

The temperature on the wall of the borehole T_{w} can be determined from the solution of the heat conduction problem in the rock formation, which is given by Eqs. (2)–(5).

The exact solution T_{r} of Eqs. (2)–(5) for the temperature $T_i^{\mathrm{a}} = 1$ was found by Carslaw and Jaeger [22], as

$$T_1(r,\tau) = 1 - \frac{2Bi_i}{\pi}\int_0^\infty \frac{e^{p^2\tau}}{p}\,\frac{\{J_0(pr)[pY_1(p) + Bi_i Y_0(p)] - Y_0(pr)[pJ_1(p) + Bi_i J_0(p)]\}\mathrm{d}p}{\{[pJ_1(p) + Bi_i J_0(p)]^2 + [pY_1(p) + Bi_i Y_0(p)]^2\}}, \tag{19}$$

where $J_0(p)$ and $J_1(p)$ are Bessel functions of the first kind of the order 0 and 1, respectively; $Y_0(p)$ and $Y_1(p)$ are Bessel functions of the second kind of the order 0 and 1, respectively.

For the arbitrary T_i^{a}, due to the Duhamel theorem, solution of Eqs. (2)–(5) can be presented in the following form

$$T_{\mathrm{r}} = \frac{\partial}{\partial \tau}\int_0^\tau T_i^{\mathrm{a}}(z,p)T_1(r,\tau - p)\mathrm{d}p. \tag{20}$$

The heat flux on the bore-face, q_{w}, can be readily computed from Eqs. (19) and (20), as

$$q_{\mathrm{w}} = -\frac{1}{Bi_i}\frac{\partial T_{\mathrm{r}}}{\partial r}\bigg|_{r=1}$$
$$= T_i^{\mathrm{a}}(z,\tau) + \int_0^\tau T_i^{\mathrm{a}}(z,p)\frac{\partial}{\partial \tau}q_i(\tau - p)\mathrm{d}p, \quad (0 < \tau < \tau_c), \tag{21}$$

where

$$q_i = -\frac{1}{Bi_i}\frac{\partial T_1}{\partial r}\bigg|_{r=1} = \frac{4Bi_i}{\pi^2} \times \int_0^\infty \frac{e^{-p^2\tau}dp}{\{[pJ_1(p)+Bi_iJ_0(p)]^2+[pY_1(p)+Bi_iY_0(p)]^2\}p}. \tag{22}$$

As it can be seen, the exact solution given by Eqs. (19)–(22) is rather complex. Applying the approximate generalized integral balance method, Chugunov et al. [26] proposed a simple approximate solution for the problem described by Eqs. (2)–(5), as

$$T_r = \begin{cases} T_i^a \frac{Bi_i \ln(l/r)}{1+Bi_i\ln(l)}, & r \leqslant l \\ 0, & r > l \end{cases} \tag{23}$$

$$q_w(\tau) = T_i^a q_i, \tag{24}$$

where

$$q_i = 1/[1+Bi_i\ln(l)] \tag{25}$$

and l is the so-called radius of thermal influence, which is defined as

$$l = 1 + \frac{2.084+0.704Bi_i}{1.554+0.407Bi_i}\sqrt{\tau}. \tag{26}$$

Comparison with an exact solution (19)–(22) proves the sufficient accuracy of the approximate solution (23)–(26) for simulating the heat flux on the bore-face and the temperature field in the formation. In [26] it was shown that these solutions practically coincide for all values of parameter Bi_i and time τ (see Figs. 4 and 5 in [26]). Based on this approximate solution, the temperature on the well's wall within the interval $0 \leqslant z \leqslant H_0/H$ can be presented in an explicit form as follows

$$T_w = T_r|_{r=1} = T_i^a(z,\tau)(1-q_i(\tau)) \tag{27}$$

Further, denoting $\tilde{\tau} = \tau - \frac{d_r H(z-H_0/H)}{r_w^2 V}$ in the Model 2 (boundary value problem (2)–(4) and (6)), it can be easily converted to the problem (2)–(5), which was solved above. As a result, the solution of the Eqs. (2)–(4) can be presented in the following form

$$T_w = T_r|_{r=1} = T_i^a(z,\tau)\left[1 - q_i\left(\tau\frac{1-z}{1-H_0/H}\right)\right], \tag{28}$$

where $H_0/H \leqslant z \leqslant 1$ and q_i is defined by Eqs. (25) and (26).

It can be readily shown that shortly after onset of the fluid circulation in the borehole, starting from the finite time $\tau = \tau_s$, the heat flow in the borehole stabilizes and becomes quasi-steady. A series of computations performed by Raymond [12] for the unsteady model of the borehole fluid circulation indicated that shortly after the "bottom comes up" the thermal behavior of a mud system begins to approach a slow, logarithmic decline. Such decline suggests that diffusion of heat into and out of the formation is a controlling factor. In this case the unsteady-state terms in Eqs. (9) and (10) become negligible. This is quite reasonable because of the greater volume and lower thermal conductivity of the formation surrounding the borehole. In mathematical terms this means that the time of stabilization can be approximately assessed by the formula $\tau_s \sim H/v^d + H/v^a$. In a quasi-steady model the temperature is a function of time, but its derivative with respect to time is negligibly small and, therefore, can be ignored, and the dependence of time is introduced only through the formation temperature, which is presented by the approximate formulae (25)–(28). Accounting for these formulae, for the quasi-steady heat transfer regime Eqs. (9) and (10) can be reduced to

$$\frac{\partial T_i^d}{\partial z} = a_i^d(T_i^a - T_i^d) + f \tag{29}$$

$$\frac{\partial T_i^a}{\partial z} = a_i^a(T_i^a - T_i^d) + b_i T_i^a q_i[\tau\eta(z)] + f, \tag{30}$$

where the borehole depth during drilling increases with time and can be computed by equation $H = H_0 + V\tau r_0^2/d_r$ and function $\eta(z)$ is defined by the following formula:

$$\eta(z) = \begin{cases} 1, & 0 \leqslant z \leqslant H_0/H, \\ (1-z)/(1-H_0/H), & H_0/H \leqslant z \leqslant 1. \end{cases} \tag{31}$$

3. Solution and discussion of the results

In further computations it is assumed that the initial formation temperature can be approximated by the linear function of depth, i.e. $t_0 = t_f(z) = Az^* + B$, which leads to $f = -1$ in Eqs. (29) and (30). Correctness of this approximation is well documented in literature related to measurements of equilibrium formation temperature [20]. The numerical solution of this system of first-order differential equations with boundary conditions (11)–(13) does not present any difficulties. Unfortunately, in the general case Eqs. (29) and (30) cannot be solved analytically because q_i is a function of axial coordinate z. However, if there is only fluid circulation in the borehole, without drilling ($H = H_0$) (this regime is used in order to clean up the borehole and to remove the debris), then at this circulation stage of the borehole exploitation q_i is defined by Eq. (25), which depends on time only. For this particular case, Eqs. (29) and (30) possess a simple close-form solution, namely

$$T_i^d = c_i^{(1)}\exp(\lambda_i^{(1)}z) + c_i^{(2)}\exp(\lambda_i^{(2)}z) + d_i, \tag{32}$$

$$T_i^{\rm a} = c_i^{(1)}(1+\lambda_i^{(1)}/a_i^{\rm d})\exp(\lambda_i^{(1)}z) + c_i^{(2)}(1+\lambda_i^{(2)}/a_i^{\rm d}) \times \exp(\lambda_i^{(2)}z) + (d_i + 1/a_i^{\rm d}), \tag{33}$$

where

$$d_i = -(a_i^{\rm a} - a_i^{\rm d} + b_i q_i)/(a_i^{\rm d} b_i q_i), \tag{34}$$

$$\lambda_i^{(1,2)} = 0.5\left[(a_i^{\rm a} - a_i^{\rm d} + b_i q_i) \pm\sqrt{(a_i^{\rm a} - a_i^{\rm d})^2 + 2(a_i^{\rm a} + a_i^{\rm d})b_i q_i + (b_i q_i)^2}\right], \tag{35}$$

and $i = 1, 2, 3, \ldots, N+1$.

In Eqs. (32) and (33) the coefficients of integration $c_i^{(1)}$ and $c_i^{(2)}$ can be readily obtained by satisfying the boundary conditions (11)–(13). Since the only reason why the solution of Eqs. (29) and (30) cannot be obtained analytically in the form of formulae (32), (33) is the dependence of the heat flux q_i on variable z, it would be interesting to try to replace function $q_i\,[\tau\eta(z)]$ in Eq. (30) with its mean value averaged over coordinate z, namely

$$\bar{q}_i(\tau) = \int_0^1 q_i[\tau\eta(z)]{\rm d}z. \tag{36}$$

Fig. 2 illustrates time variations of the function $q_i[\tau\eta(z)]$ for different z and Bi. Even for relatively low Biot numbers (e.g. $Bi = 1$) for different z, the discrepancy of the function $q_i[\tau\eta(z)]$ from its mean value $\bar{q}_i(\tau)$ is relatively small. For the bigger values of Biot number ($Bi > 5$) the plots of $\bar{q}_i$ and q_i practically coincide. Hence, the mean function $\bar{q}_i(\tau)$ can be used in Eq. (30) as a quite accurate approximation of $q_i[\tau\eta(z)]$. In this case, the solution of Eqs. (29) and (30) has the same form as the obtained above solution (32), (33), where the function q_i should be simply replaced by $\bar{q}_i$ defined by equation (36). Finally, substituting solution (32) and (33) (where q_i is replaced by $\bar{q}_i$) into the boundary conditions (11)–(13), the following system of linear algebraic equations for unknown constants $c_i^{(1)}$ and $c_i^{(2)}$ is obtained

$$c_1^{(1)} + c_1^{(2)} = T_{\rm in} - \bar{d}_1 \tag{37}$$

$$c_i^{(1)}\exp(\lambda_i^{(1)}z_i) + c_i^{(2)}\exp(\lambda_i^{(2)}z_i) - c_{i+1}^{(1)}\exp(\lambda_{i+1}^{(1)}z_i) - c_{i+1}^{(2)}\exp(\lambda_{i+1}^{(2)}z_i) = \bar{d}_{i+1} - \bar{d}_i \tag{38}$$

$$c_i^{(1)}a_{i+1}^{\rm d}\lambda_i^{(1)}\exp(\lambda_i^{(1)}z_i) + c_i^{(2)}a_{i+1}^{\rm d}\lambda_i^{(2)}\exp(\lambda_i^{(2)}z_i) - c_{i+1}^{(1)}a_i^{\rm d}\lambda_{i+1}^{(1)}\exp(\lambda_{i+1}^{(1)}z_i) - c_{i+1}^{(2)}a_i^{\rm d}\lambda_{i+1}^{(2)}\exp(\lambda_{i+1}^{(2)}z_i) = a_i^{\rm d} - a_{i+1}^{\rm d} \tag{39}$$

$$c_{N+1}^{(1)}\lambda_{N+1}^{(1)}\exp(\lambda_{N+1}^{(1)}) + c_{N+1}^{(2)}\lambda_{N+1}^{(2)}\exp(\lambda_{N+1}^{(2)}) + 1 = 0 \tag{40}$$

where $i = 1, 2, 3, \ldots, N$ and $\bar{d}_i = -(a_i^{\rm a} - a_i^{\rm d} + b_i\bar{q}_i)/(a_i^{\rm d} b_i \bar{q}_i)$, and

$$\bar{\lambda}_i^{(1,2)} = 0.5\left[(a_i^{\rm a} - a_i^{\rm d} + b_i\bar{q}_i). \pm\sqrt{(a_i^{\rm a} - a_i^{\rm d})^2 + 2(a_i^{\rm a} + a_i^{\rm d})b_i\bar{q}_i + (b_i\bar{q}_i)^2}\right]. \tag{41}$$

This system of $2(N+1)$ linear equations regarding $2(N+1)$ unknown constants $c_i^{(1)}$ and $c_i^{(2)}$ is solved by a sweep method.

For better graphical illustration of numerical results, the equilibrium temperature in formation, t_0, and temperatures in the borehole, $t^{\rm a}$ and $t^{\rm d}$, are represented by the following non-dimensional functions

$$U_0 = \frac{t_0 - B}{AH} = z, \quad U^{\rm a} = T^{\rm a} + U_0, \quad U^{\rm d} = T^{\rm d} + U_0 \tag{42}$$

The influence of fluid loss through the fracture on a temperature distribution in the borehole is illustrated

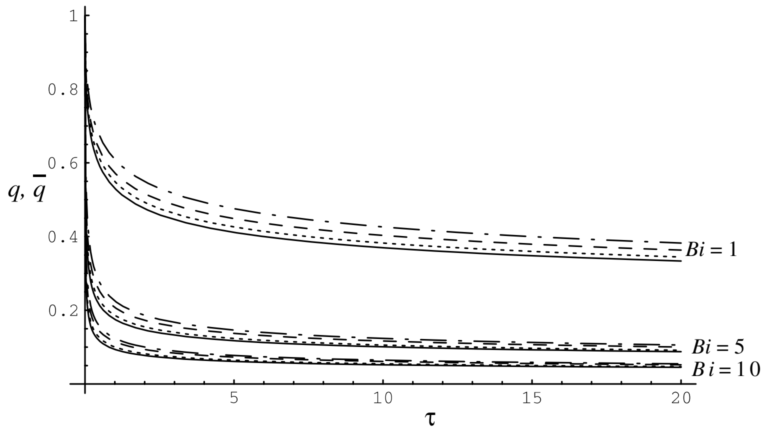

Fig. 2. Variation of the bore-face heat flux $q[\tau\eta(z)]$ computed by Eqs. (25), (26) and (31) and its mean value $\bar{q}$ with respect to time for different z and Bi during drilling to the depth $H = 2000$ m starting from the depth $H_0 = 1000$ m. Solid line—mean value $\bar{q}$; dotted line—solution q for $z = 0.2$; dashed line—solution q for $z = 0.6$; dash-dotted line—solution q for $z = 0.8$.

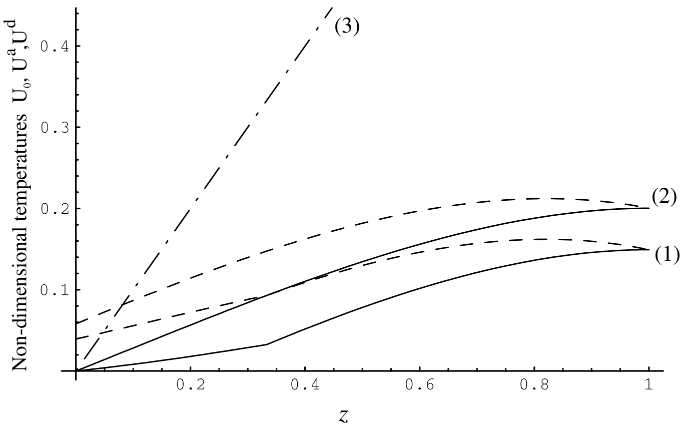

Fig. 3. Temperature distribution in the borehole for drilling velocity $V = 0.04$ m/s, injection flow rate $G = 30$ kg/s and drilling time 70 h. Solid lines—temperature in a drilling pipe U^d; dashed line—temperature in the annulus U^a; dot-dash line—equilibrium temperature U_0. (1)—fluid loss through the single located at $z = 1/3$; (2)—no fluid loss; (3)—equilibrium temperature.

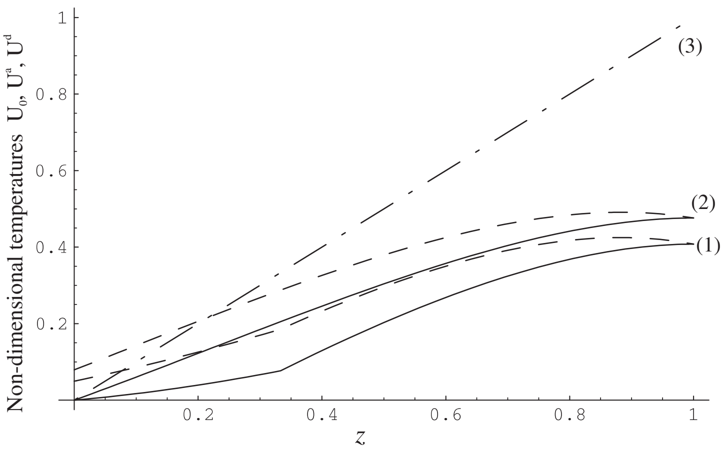

Fig. 4. Temperature distribution in the borehole for drilling velocity $V = 0.04$ m/s injection flow rate $G = 10$ kg/s and drilling time 70 h. Solid lines—temperature in a drilling pipe U^d; dashed line—temperature in the annulus U^a; dot-dash line—equilibrium temperature U_0. (1)—fluid loss through the single located at $z = 1/3$; (2)—no fluid loss; (3)—equilibrium temperature.

in Figs. 3 and 4. Even though the derived above model allows computing the temperature distribution for the case of several fractures intersecting the borehole, obviously, considering a particular case when the fluid loss occurs through a single major fracture that intercepts the borehole can readily elucidate this effect. In numerical computations presented in Figs. 3 and 4 it is assumed that the fracture intersects the borehole at the depth of 1000 m ($z = 1/3$) and the borehole is drilled to the depth of 3000 m ($z = 1$) starting from the point $H_0 = 1000$ m. In Fig. 3 the injection flow rate through the drilling pipe, G, is equal to 30 kg/s, half of the injected water flows away from the borehole through the fracture and half returns to the surface so that the outflow rate is 15 kg/s. In Fig. 4 the injection flow rate through the drilling pipe, G, is equal to 10 kg/s, half of the injected water flow goes away from the borehole through the fracture and half returns to the surface so that the outflow rate is 5 kg/s. As it can be seen, temperatures in the borehole are significantly affected by the existence of fractures that intercept the borehole and through which the injected fluid flows away into the rock formation. The fluid loss of the circulating fluid leads to reducing the borehole temperature since the flow rate in the upper part of the annulus is lower and it decreases the intensity of heat transfer (smaller h^w and h^a) near the borehole mouth. The comparison of Figs. 3 and 4 demonstrates that for the smaller injection flow rate, the borehole fluid has higher temperatures. For instance, as it can be seen, the triple reduction of the flow rate leads to a double increase of the fluid temperature. Although for a smaller flow rate the heat transfer is less intensive, the longer time during which the injected fluid remains in the borehole, and therefore is subjected to longer heating, has more profound effect on the fluid temperature.

The mathematical model discussed in this paper that accounts for the fluid loss through the discrete fractures, which intercept the well, can be readily applied for computing the borehole temperatures without the fluid loss (this case is denoted by (1) in Figs. 3 and 4). The above results can be used for validation of the proposed model by comparing these curves with numerical results obtained in the previous studies of the borehole temperatures distribution without the fluid loss. For example, results available in [10–15] are in good agreement with the solutions presented in Figs. 3 and 4.

4. Conclusions

The following conclusions are drawn:

1. A mathematical model for the temperature distribution in the drilling borehole that accounts for the fluid loss through discrete fractures that intercept the borehole is proposed.
2. The model is simplified by applying the accurate explicit formula for the heat flux on the bore-face instead of using a numerical finite difference solution or a complex exact analytical solution found by Laplace transform.
3. A closed-form approximate solution for the temperature distribution in a borehole is obtained. This solution can be used for analyzing the effect of different parameters on the borehole temperature. Its accuracy can be validated by a comparison with solutions available in literature for the "no fluid loss" case.
4. The fluid leakage through the fracture intercepting the borehole leads to the reduction of the borehole temperature. This fact is beneficial for the exploitation of the drilling bit within the high-temperature rock formation [5] when the efficiency of the cooling system is of major concern. This effect (reduction of the borehole temperature) can be also reached by increasing the injection flow rate.

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